
Conditions of the Competitive Exclusion Principle in a Varying Environment for a Class of Lotka–Volterra Models

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Understanding how environmental variability affects species coexistence is a central question in ecology. In this work, we propose a two-species Lotka–Volterra competition model where resources change abruptly between two levels, representing sudden environmental transitions. Using averaging theory for non-smooth dynamical systems with rapid periodic switching, we derive explicit theoretical and general conditions under which species coexist or one excludes the other. This approach leads to stability results that corroborate and extend the findings on an autonomous Lotka–Volterra model with resource-dependent competition [1].

Our results show that a species excluded in both constant environments can win or coexist under periodic fluctuations at low resource levels; alternatively, each species may win in one environment after periodic fluctuations during which they coexist. These findings highlight how temporal resource variability can promote coexistence even when one species dominates in each constant environment. The resource bifurcation values Z_{ci} and Z_{cj} derived here are structurally analogous to Tilman's R^* values [4], and the coexistence mechanism is shown to be a form of temporal averaging distinct from Chesson's storage effect [2, 3].

Keywords: Lotka–Volterra competition model; averaging theory; environmental fluctuations; competitive exclusion principle; coexistence; resource switching; bifurcation.

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References

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