Colloque International du Réseau Équations aux Dérivées Partielles, Modélisation et Contrôle (Réseau EDP-MC)

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Mountain pass solutions for nonlinear elliptic PDEs with curve singuralities on the boundary.

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Let $\Omega \subset \mathbb{R}^{N+1}$, $N \geq 2$ be a bounded domain with smooth boundary $\partial \Omega$ and let $\Sigma \subset \partial \Omega$ be a closed curve containing the origin. We are interested in the existence of positive solutions of mountain pass type in $H^1(\Omega)$ for the following Hardy-Sobolev trace-type boundary value problem

$$\begin{cases} -\Delta u + u = 0, & \text{in } \Omega \\ \frac{\partial u}{\partial \nu} = \lambda \rho_{\Sigma}^{-s_1} u^{q_{s_1} - 1} + \rho_{\Sigma}^{-s_2} u^{q_{s_2} - 1}, & \text{on } \partial \Omega \end{cases}$$

where λ is a real parameter, ν denotes the outward unit normal to $\partial\Omega$ and $\rho_{\Sigma}:\partial\Omega\to\mathbb{R}$ is the distance function in the induced metric \tilde{g} on $\partial\Omega$ to the curve Σ defined by

$$\rho_{\Sigma}(x) := \inf_{y \in \Sigma} d_{\tilde{g}}(x, y).$$

The parameters s_1 , s_2 satisfy $0 < s_2 < s_1 < 2$, and the corresponding critical Hardy-Sobolev exponents are given by

$$q_{s_i} := \frac{2(N - s_i)}{N - 1}$$
, for $i = 1, 2$.

Our goal is to prove the existence of positive solutions obtained via the mountain pass theorem, under suitable assumptions on the local geometry of $\partial\Omega$ and the curve Σ .